Int. J. Heat Mass Transfer. Vol. 16, pp. 1467–1476. Pergamon Press 1973. Printed in Great Britain

EXPERIMENTAL STUDY OF THE STABILITY OF DIFFERENTIALLY HEATED INCLINED AIR LAYERS

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(*Received* 31 *May* 1972 *and in revised form* 24 *November* 1972)

Abstract—This paper reports the experimental determination of the critical Rayleigh numbers governing the stability of horizontal, vertical and inclined air layers in the conduction regime. The experimental method consists of measuring the heat flux in the immediate neighbourhood of the instability and extrapolating this data to the state of pure conduction. The measured critical Rayleigh number for the horizontal case is within 1 per cent of the accepted value (1708). In the vertical case, the measured value of the critical Grashof number is 11 000 ± 510, which is in very close agreement to the value of 11 024 predicted by Unny. In the inclined case results are presented at angles from the horizontal 15, 30, 45, 60, 75, 80 and 85 degrees. The results are in agreement with the predictions of Unny and Hart to within a maximum deviation of about 20 per cent.

The Nusselt number in the neighbourhood of the instability is also reported.

NOMENCLATURE

A, aspect ratio of inclined slot $= H/L$ [dimensionless];
C_p, specific heat of air [Btu/lb$_m$F];
e, emf of heat flux meter [mV];
Gr, Grashof number $= Ra/Pr$ [dimensionless];
Gr_c, critical Grashof number [dimensionless];
g, acceleration of gravity [ft/h^2];
H, length of inclined slot, as defined in Fig. 1 [ft];
K, constant in equation (5) [dimensionless];
K_1, K_2, constants in equations (6) and (7) respectively [dimensionless];
k, thermal conductivity of air [Btu/ft h F];
L, width of tilted slot, defined in Fig. 1 [ft];
Nu, Nusselt number $= q_a L/\Delta T k$ [dimensionless];
Pr, Prandtl number of air;
p, pressure of air [lb$_f$/ft^2];
q, total heat flux from lower to upper plate $= q_r + q_a$, [Btu/ft^2 h];
q_a, heat flux transported across air layer by conduction and natural convection [Btu/ft^2 h];
q_r, heat flux transported across air layer by radiation [Btu/ft^2 h];
q_E, rate of electrical heating supplied to heat plate, expressed as a flux [Btu/ft^2 h];
R, gas constant for air [ft lb$_f$/R];
Ra, Rayleigh number;

$$= \frac{g\rho^2 \beta \Delta T L^3 C_p}{\mu k} \text{ [dimensionless]};$$

Ra_c, critical Raylelgh number;
T, absolute temperature of air [R];
ΔT, temperature difference applied across the air layer (Fig. 1) [F];
x, y, z, co-ordinates, defined in Fig. 1 [ft];
α, calibration constant of heat flux meter [Btu/ft^2 h mV];

β, volumetric thermal expansion coefficient of air [R^{-1}];
θ, angle of tilt of air layer, defined in Fig. 1 [degree];
μ, viscosity of air [lb_f s/ft^2];
ρ, density of air [lb_m/ft^3].

INTRODUCTION

THE PROBLEM of stability and convective motion of a horizontal fluid layer heated from below has gained a considerable and growing attention over the last two decades; so has the problem of stability and convective motion in a long vertical slot heated from one side. However, very little has been done to bridge the gap between these two separate problems, although, if one considers the general case of a differentially heated, inclined fluid layer (Fig. 1), they are each special cases of a single problem.

In the first of the above-mentioned problems (the Benard problem) [1], the instability is associated with what may be called a "top-heavy" situation. For the Rayleigh number less than critical ($Ra < 1708$) the fluid is completely stationary and heat is transferred across the layer by the conduction mechanism alone. The type of motion in the neighbourhood of the instability has been subject of considerable study but is now generally thought to be roll-like [2]. Once convective motion has begun, heat transfer across the fluid layer increases.

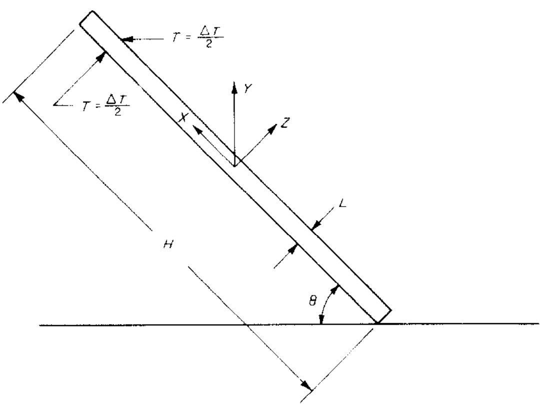

FIG. 1. Sketch of an inclined fluid layer.

In the second problem—the long vertical slot heated from one side—there exists (as opposed to the first problem), fluid motion for any finite Rayleigh number. However, provided the Rayleigh number is sufficiently small the motion is relatively simple, consisting of one large cell which fills the whole slot, the fluid rising on the hot wall, falling down the cold wall and turning at the opposite ends of the slot. This flow, first described by Batchelor [3] will be called here the "base-flow". Except in the extreme regions near the ends of the slot where the fluid turns, the base-flow vertical velocity profile is cubic, and the horizontal velocity is zero. No convective heat transfer is caused by this motion and heat is transferred across the gap by conduction only. The stability of this base flow has been analyzed in a number of studies [4–9]. The class of instability is physically different from that applying in the Benard problem: the instability appears to be brought on by the hydrodynamic breakdown on the base flow as opposed to any top-heavy situation. Vest and Arpaci [7] found that the critical condition is dependent only on the Grashof number and independent of the Prandtl number, and gave a critical Grashof number of 7880. All other studies have found a slight but significant dependence of the critical Grashof number on the Prandtl number. For $Pr = 0{\cdot}71$ all studies obtain a critical Grashof number of approximately 8000 except Unny [8] who obtained 11 024.

It is of importance at this point to indicate what aspect ratio constitutes a "long" vertical slot for the purposes of the present study. A classification of the regimes of instability in a vertical slot has been given by Gill and Davey [10]. They concluded that analyses based on assuming the aspect ratio is infinity (i.e. assuming the base flow defined earlier) are valid provided that the aspect ratio is greater than a certain value, which depends on the Prandtl number. The region on Rayleigh number vs aspect ratio plane where this assumption is valid is called the conduction regime. This regime is bounded, according to Gill and Davey by the relation:

$$\log_{10}(Gr_c\,Pr/A) < 2{\cdot}5. \quad (1)$$

The critical Grashof number, as obtained by Rudokov, Unny or Vest and Arpaci is approximately 10^4 so that to ensure the base flow is in the conduction regime: we must have, approximately,

$$A > 30 \cdot Pr. \quad (2)$$

A long slot, for the purposes of this paper, is defined as one satisfying this inequality.

The more general case of a tilted, differentially heated from below, long fluid slot or layer (Fig. 1) has been analyzed by Gershuni and Zhukhovitskii [6], Unny [8], Hart [9], Liang and Acrivos [11] and Birikh *et al.* [12]. As in the case of the vertical slot, provided the fluid layer has a finite angle of tilt, θ, there exists a base flow for any finite Rayleigh number. The tilted layer is therefore in general subject to two types of instability, the static top-heavy type instability associated with the horizontal (Benard) problem, and the dynamic type which applies the vertical slot. At low angles of tilt the former type comes into play first, and at angles of tilt near the vertical and beyond, the latter type. At some intermediate angle, θ_c there is a crossover from one class of instability to the other. For $\theta < \theta_c$ the instability is characterized by a critical Rayleigh number, independent of the Prandtl number, and given by:

$$Ra_c \cos\theta = 1708. \quad (3)$$

For $\theta > \theta_c$ it is characterized by a critical Grashof number, largely independent of the Prandtl number. The angle θ_c is therefore a strong function of the Prandtl number. The type of flow at the point of instability is roll-like with axis (referring to Fig. 1) in the x-direction for $\theta < \theta_c$ and in the y-direction for $\theta > \theta_c$. The crossover angle, θ_c, for air ($Pr \doteq 0{\cdot}71$) is given by Hart to be 72°, and by Unny to be 78°.

Although there are ample experimental verifications of the critical condition for the horizontal layer, there are very few for the vertical and inclined layer, particularly if one restricts oneself to layers that are long by the criterion of equation (2). Vest and Arpaci [7] determined Gr_c for a long vertical air layer using a flow visualization technique and obtained $Gr_c = 8600 \pm$ 10 per cent. The recent experimental results of Hart [9], although strictly not carried out on layers which were long on the basis of equation (2), are extensive and important to the present study. Using water ($Pr \doteqdot 6{\cdot}7$) as the fluid, layers with aspect ratios of 25 and 37, and a flow visualization technique, his experiments confirm his theoretical stability analysis (which covers both long and moderate aspect ratio slots), for $\theta < \theta_c$. However, for $\theta > \theta_c$, i.e. near the vertical, measured critical Rayleigh numbers are from 17 to 80 per cent larger than predicted. Hart puts this discrepancy down as being due partly to the experimental difficulty of detecting very weak flows, and partly due to the non-parallel nature of the base flow.

This paper presents an experimental determination of the critical Rayleigh number for angles of tilt, θ from 0 to 90°. The fluid used is air ($Pr \simeq 0{\cdot}72$) and the aspect ratio is 44; the slot is consequently assumed "long", and the conduction regime is therefore assumed to prevail throughout the study.

Briefly stated, the experimental method consisted of maintaining a constant temperature difference and spacing across the tilted air layer, and passing through the point of instability by slowly increasing the air pressure by steps, measuring the heat flux at each step. As has been pointed out, in the "base flow" region below the critical condition, heat is transferred across the gap by conduction alone, and hence the measured heat flux is invariant with pressure (Rayleigh number) until the critical condition is reached. From this point onward, subsequent increases in pressure produce increases in the measured heat transfer. By plotting the data in the neighbourhood of the critical condition, it is possible to arrive at a close estimate of the critical condition. A similar method was used by Thompson and

Sogin [13] for a horizontal gas layer. The heat flux was measured by a combination of the heat flux meter and guarded hot plate principles, permitting an absolute determination (as opposed to the Thompson–Sogin method). This paper therefore reports both the measured critical Rayleigh numbers, and the Nusselt number in the neighbourhood of the critical condition.

DETAILS OF THE EXPERIMENT

Figure 2 shows a sketch of the apparatus. It consists essentially of two parallel copper plates, each 22 in. by 24 in. by 3·8 in. thick mounted in a pressure (or vacuum) vessel whose pressure can be varied from a few mm Hg up to 100 psia. The plates are maintained at uniform (but different) temperatures by means of two water streams passing through copper tubes soldered to the back of the plates. The warm stream (~80F) passing through the lower plate is maintained at constant temperature by recirculation through a constant temperature bath. The cool stream (~60F) is drawn from the main-water line and allowed to run to drain. The whole system (including the pressure vessel) can be rotated about a horizontal axis so as to produce any desired degree of tilt of the two plates. The spacing between the plates can be adjusted from 0 to 5 in. and maintained constant over the plate surfaces to within $\pm 0{\cdot}0025$ in. The air space between the two plates is closed at the periphery by means of aluminium foil attached from one plate to the other. This foil maintains a linear temperature rise from the cold to hot plate.

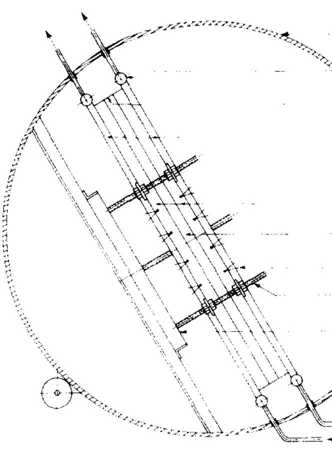

FIG. 2. The experimental apparatus.

The temperature difference between the two plates were determined from eight copper-constantan thermocouple junctions, four imbedded in each of the copper plates and connected in thermopile. The mean air temperature for evaluation of the fluid properties of air was determined as the average of two thermometers, one in each of the circulating water lines. The air pressure was determined from two precision pressure gauges, one reading from 0 to 15 psia, the other from atmospheric pressure to 100 psia. The spacing between the plates was determined from telescoping gauges and a micrometer. Further details of the apparatus may be found in [14] and [15].

The method of measuring the heat flux is sketched in Fig. 3. A recess 5 in. by 5 in. by $\frac{1}{4}$ in. deep machined into the lower plate contains a heat flux meter and a $\frac{1}{4}$ in. copper plate (heater plate) in which a nichrome resistance wire of known resistance is imbedded. The heat flux meter consists of a polyvinylchloride disc $\frac{1}{8}$ in. thick by 4 in. dia with approximately 600 thermocouples connected in thermopile and woven into the disc so as to measure the temperature difference across it. The principle of

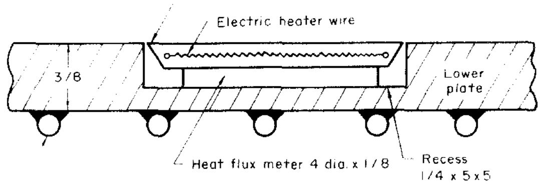

FIG. 3. Sectional view of lower plate showing assembly of heater plate and heat flux meter in recess (not to scale).

the method used is the same as the guarded hot plate principle in that when the heat current has been varied until the heater plate is at the same temperature as the lower plate, all of the electrical heating supplied to the heat plate goes into the air layer above, and hence the heat flux through the air layer can be determined directly. The condition for equality of temperature of the heater plate and lower plate is obviously met when the heat flux meter emf is zero. By using the heat flux meter to determine the condition for equality of the two temperatures rather than say, a thermocouple imbedded in each plate, a much higher degree of sensitivity is gained in the heat flux measurement.

In practice the heater current was not adjusted until the heat flux meter emf was exactly zero since due to the thermal inertia of the system, this required too long a time. Rather the current was adjusted until the heat flux meter emf was within $\pm 1 \cdot 2$ mV of zero corresponding to a temperature difference between the heater plate and the lower plate of $\pm 0 \cdot 1$ F°. The heat flux into the air layer q was then determined from:

$$q = q_E + \alpha e$$

where q_E is the electrical heating supplied to the heater plate, e is the heat flux emf and α is a constant; this assumes that the heat transfer from the heater plate to the lower plate is proportional to the heat flux meter e.m.f. The quantity α, the calibration constant of the heat flux meter *in situ* was determined by a calibration procedure which involved setting $q = 0$ by adjusting upper and lower plate to the same temperature and by varying the heater current obtain a graph of q_E vs e, the slope of this graph being α. (Of course once α had been determined, q could have been measured by having no heater current ($q_E = 0$), measuring e, and using the resultant equation, $q = \alpha e$. However, such a procedure would have resulted in a significant temperature difference between the heater and lower plates. Since an important requirement of the experimental apparatus was an isothermal lower boundary surface for the air layer such a situation was obviously unacceptable.)

The experimental procedure was started by setting the angle of tilt of the plates, flushing the pressure vessel with dry air, drawing a vacuum of approximately 1 mm Hg and taking a complete set of measurements. The pressure was then raised to about 80 per cent of the calculated critical pressure corresponding to the estimated critical Rayleigh number for the angle under test, on the basis of Unny's predictions. Although this involved a considerable change in pressure, without exception, no significant change in the heat flux was observed. The pressure was then increased approximately in 5 per cent steps, readings being taken at each step. At one step in this process, the heat flux, having previously remained constant, independent of the pressure, would increase, corresponding to the critical condition. Subsequent increases in pressure would then result in increased heat transfer.

In processing the data, it was necessary to subtract from the measured heat transfer the radiative heat exchange between the plates. This was determined from the very first reading taken at approximately 1 mm Hg. At this condition, the air layer was known to be stagnant. From the known thermal conductivity of air, the component of this low pressure heat transfer associated with conduction through the air could be calculated and subtracted from the total low pressure heat transfer, leaving only radiation heat transfer. The latter was assumed constant with respect to pressure and subtracted from all subsequent heat transfer measurements in that run. The radiation correction never involved more than 10 per cent of the total heat transfer.

Assuming air obeys the ideal gas law, the Rayleigh number can be written as:

$$Ra = \frac{p^2 \Delta T g L^3 C_p}{R^2 T^3 k\mu}.$$

Over the pressure range of interest μk and C_p are independent of pressure. They were evaluated, using the data of Glassman and Bonilla [16], at the mean of the two plate temperatures; the latter was also used for the evaluation of T.

As a preliminary check on the apparatus, the thermal conductivity of air was measured using the apparatus. The heat flux was measured at low pressure for various plate spacings and the resultant heat transfer coefficients were plotted against the inverse of the spacing. The thermal conductivity, as determined from the slope of this graph, was within 1 per cent of the literature value. The details are given in [14].

RESULTS FOR HORIZONTAL CASE

The behaviour of the heat transfer, for the case of a horizontal fluid layer, in the immediate vicinity of the critical condition is known on theoretical grounds to be given by:

$$Nu = 1 \qquad Ra < Ra_c \tag{4}$$

$$Nu = 1 + K\left(1 - \frac{Ra_c}{Ra}\right) \qquad Ra > Ra_c \tag{5}$$

where K is a constant. (The latter equation was first given by Malkus and Veronis [17], and has been shown to be exact in the limit $Ra \to Ra_c + 0$ by Platzman [18].) Therefore, if one plots the experimental points in the immediate vicinity of the critical condition in the form $Nu - 1$ vs $1/Ra$ the result should be a straight line which intercepts K and $1/Ra_c$. In this manner the values of K and Ra_c were determined for the horizontal layer in four independent experiments using plate spacings of 0·5, 0·75, 1·0 and 2·0 in. respectively. Figure 4 shows the plot for the case where the spacing was 0·75 in. and Table 1 summarizes the results for all four spacings. The value of Ra_c and K were arrived at by the best (least square) fits to the data points. The theoretical value of Ra_c is, of course, 1708, and for K, 1·44(5) (Platzman [18] and Hollands [19]). These values are within 1 and 3 per cent of the average of the experimental values respectively. The results are one of the most precise experimental determinations of Ra_c reported to date and present a good experimental verification of equation (5). (The data of Schmidt and Saunders [20] have also been shown to be in agreement with this equation [19].) These results also represent an excellent preliminary check on the apparatus before entering into the tilted and vertical layer determinations.

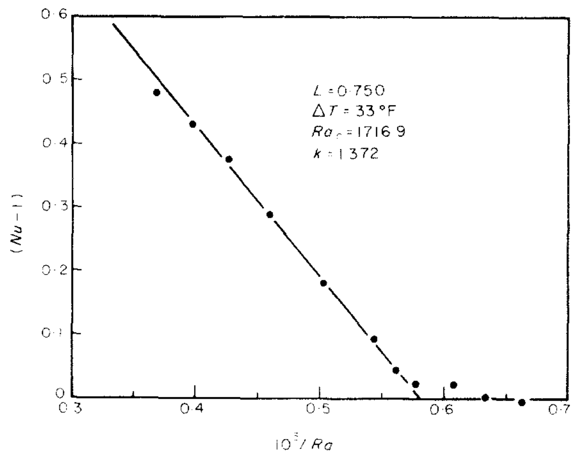

FIG. 4. Plot of heat transfer data in vicinity of critical condition for horizontal air layer; also shown in best straight line-fit to the data.

Table 1. Measured values of Ra_c *and* K *for horizontal case*

Spacing L (in.)	Temperature difference, ΔT (deg F)		Ra_c	K
0·500	27		1718	1·305
0·750	33		1717	1·372
1·000	20		1738	1·398
2·000	34		1725	1·499
		Average	1724·5	1·391
		Theoretical	1708	1·445

RESULTS FOR TILTED AND VERTICAL CASES

Data were obtained near the critical condition for angles of tilt, (angle θ in Fig. 1) of 15, 30, 45, 60, 75, 80, 85 and 90 degrees. All data were obtained using a plate spacing of 0·50 in. and a temperature difference of 20F. Figure 5 shows the results plotted in the form $Nu - 1$ vs $1/Ra$, the same form used in the horizontal case.

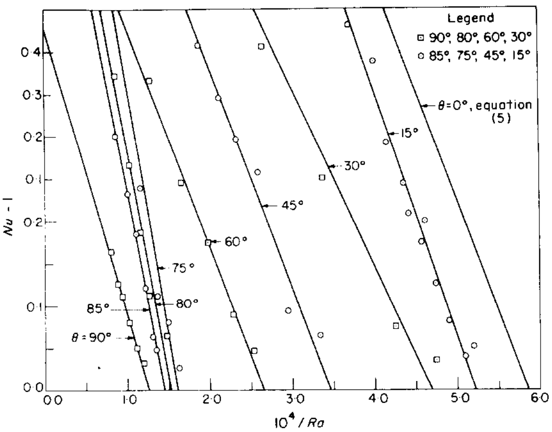

FIG. 5. Plot of heat transfer data in vicinity of critical condition for inclined air layers; also shown is best straight line fit to the data for each angle, θ.

As in the horizontal case, the best (least square) straight line was passed through the data, so as to fit an equation of the form of equation (5). The resultant values of Ra_c and K are given in Table 2 which also gives the uncertainties in Ra_c as determined in the usual way from least-square analysis of data on a 95 per cent confidence basis.

Some difficulties were experienced in measuring the heat transfer in the inclined position, particularly at angles of 30° and 45°, and this helps to explain why there is greater uncertainty in the values of Ra_c in the tilted case compared to the horizontal case. The difficulty arose out of the unsteady nature of the heat transfer which was observed at pressures slightly greater than critical. The heat flux, as measured by the heat flux meter, (whose e.m.f. was displayed on a millivolt recorder) performed an oscillation with a period which varied, but which was of the order of minutes. The heat fluxes were averaged over a period of approximately 15 min and it is these average values which are reported in the Nusselt numbers plotted in Fig. 5. This averaging process was subject to some error and the values of the Nusselt number therefore have a greater scatter than in the horizontal case.

Table 2. Measured values of Ra_c *and* K *for tilted case, (based on fitting data to equation* (5)

Angle θ (deg)	Ra_c	K	$K\,Ra_c$
15	1911 ± 49	1·515	2900
30	2126 ± 194	0·987	2100
45	2890 ± 183	0·911	2640
60	3761 ± 219	0·687	2582
75	6215 ± 487	0·952	5920
80	6547 ± 761	0·821	5380
85	6942 ± 178	0·760	5270
90	7861 ± 363	0·431	3397

The fitting of straight lines to the data of Fig. 5 implicitly assumed that the behaviour of the heat transport near the instability is given by equation (5). Although the data appear to fit a straight line reasonably well, this assumption may not in fact be valid for the tilted case, and may produce an error in the estimation of the Rayleigh number; (the actual behaviour of the heat transport near instability is not known for the tilted case). To investigate this possibility, the following straight lines were fitted to the same data as in Fig. 5,—a linear fit:

$$Nu - 1 = K_1\left(\frac{Ra}{Ra_c} - 1\right) \tag{6}$$

and a logarithmetic fit:

$$Nu - 1 = K_2 \log\left(\frac{Ra}{Ra_c}\right) \tag{7}$$

and the resultant values of Ra_c so determined; they are given in Table 3, with their corres-

Table 3. Dependence of the critical Rayleigh number (as measured) on the equation used to fit the data near the instability

Angle θ (deg)	Ra_c as estimated using: Equation (5) (inverse fit)	Equation (6) (linear fit)	Equation (7) (log fit)
15	1911 ± 49	1885 ± 51	1900 ± 55
30	2126 ± 194	1865 ± 404	2041 ± 196
45	2890 ± 183	2461 ± 625	2742 ± 302
60	3761 ± 219	3239 ± 219	3578 ± 302
75	6215 ± 487	5561 ± 653	5990 ± 506
80	6547 ± 761	5974 ± 562	6330 ± 833
85	6942 ± 178	6454 ± 819	6750 ± 383
90	7861 ± 363	7370 ± 365	7664 ± 446

ponding uncertainties for the same (95 per cent) confidence limits. Considerable (up to 15 per cent) differences are to be seen depending on the assumed fit. However, the uncertainty in all but one case, is less for the inverse fit (equation (5)) than for the other two fits, indicating that the data agree with this fit best. In comparing the measured critical Rayleigh numbers with theoretical, therefore the values based on the inverse fit will be used.

Figure 6 shows the comparison of these measured critical Rayleigh numbers with those predicted by Unny [8] and Hart [9] for $Pr = 0{\cdot}71$. The general form of the dependence of Ra_c on θ predicted by these works agrees well with the data. In particular there appears to be an abrupt change in the trend of the data at approximately 75°, indicating a changeover from one class of instability to another, as predicted. The data for angle θ greater than the changeover angle are bracketed above and below by the predictions of Unny and Hart.

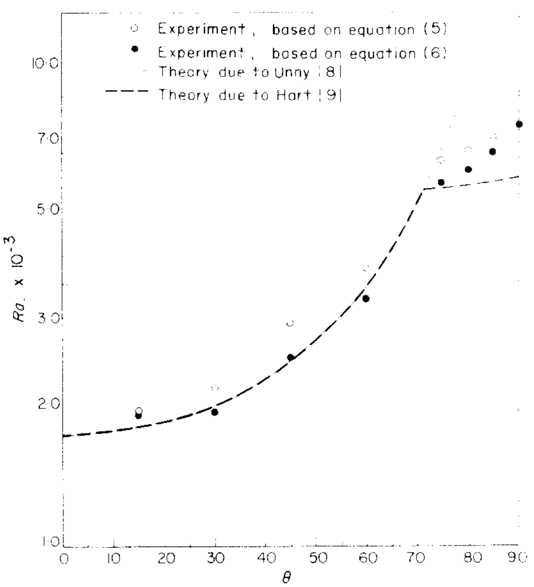

FIG. 6. Comparison of experimentally determined critical Rayleigh numbers with those predicted by Unny [8] and Hart [9].

In the case of the vertical layer ($\theta = 90°$) the results are much closer to the predictions of Unny than for Hart and other workers [5–9]; the experimental value Gr_c is $11\,000 \pm 510$, which compares favourably with Unny's value of 11 024. However, it should be noted that Hart's experimental critical Rayleigh numbers were also greater than those predicted by his analysis.

With respect to the heat transfer it is of interest to note in Fig. 5 that the changeover at 75° shows up as a change in slope of this graph as well. The fitted straight lines through the data for $\theta < 75°$ are all approximately parallel to the theoretical line at $\theta = 0$, i.e. to equation (5) with $K = 1{\cdot}445$ and $Ra_c = 1708$. This implies that, if the tilted data are assumed to fit equation (5), with Ra_c and K considered functions of θ, then it is found experimentally that:

$$K(\theta) \cdot Ra_c(\theta) \backsim \text{constant} = 1{\cdot}445 \times 1708$$

or

$$K(\theta) \cdot Ra_c(\theta) \backsim 2460 \qquad \theta < 60°.$$

Values of $K\,Ra_c$ for the various angles are given in Table 2.

CONCLUSIONS

1. Precise measurements have been carried out on the stability of a horizontal air layer heated from below and the results fit the equation of Malkus (equation (5)) very closely.
2. Measurements of the critical Grashof number for air in vertical slot of large aspect ratio have given a value of $11\,000 \pm 510$ which is in very close agreement with the prediction of Unny.
3. Measurements of the critical Rayleigh number of inclined air layers of large aspect ratio agree with the predictions of Unny within a maximum deviation of 21 per cent and an average deviation of 10 per cent. Slightly larger deviations are found with the predictions of Hart. Their predictions are recommended for engineering calculations.
4. For angles of tilt θ, of 60° or less the heat

transfer in the neighbourhood of the instability can be represented approximately by equation (5) with $K = 2460/Ra_c$.

ACKNOWLEDGEMENTS

This work was supported by a grant from the National Research Council of Canada. The authors are indebted to Dr. T. E. Unny for suggesting the problem and for many helpful discussions.

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ETUDE EXPERIMENTALE DE LA STABILITE D'UNE COUCHE D'AIR INCLINEE ET DIFFERENTIELLEMENT CHAUFFEE

Résumé—Cet article décrit la détermination expérimentale des nombres critiques de Rayleigh marquant la stabilité de couches d'air horizontales, verticales ou inclinées, dans le régime de conduction. La méthode expérimentale consiste en une mesure du flux thermique au voisinage immédiat de l'instabilité et en une extrapolation de ce résultat à l'état de conduction pure. Le nombre critique de Rayleigh mesuré pour le cas horizontal approche à un pour cent la valeur acceptée (1708). Dans le cas vertical, la valeur mesurée du nombre critique de Grashof est de 11 000 $\pm$ 510 ce qui est en très bon accord avec la valeur de 11 204 estimée par Unny. Dans le cas incliné, les résultats sont présentés pour des angles égaux, par rapport à l'horizontale, à 15, 30, 45, 60, 75, 80 et 85 degrés. Les résultats concordent avec les estimations de Unny et Hart avec un écart maximal de 20 pour cent. On donne aussi le nombre de Nusselt au voisinage de l'instabilité.

EXPERIMENTELLE UNTERSUCHUNG DER STABILITÄT EINER UNTERSCHIEDLICH BEHEIZTEN, GENEIGTEN LUFTSCHICHT

Zusammenfassung—Es wird über die experimentelle Bestimmung der kritischen Rayleigh-Zahlen für die Stabilität von horizontalen, vertikalen und geneigten Luftschichten im Bereich der Wärmeleitung berichtet. Bei dieser experimentellen Methode wird der Wärmestrom in der Nähe der Stabilitätsgrenze gemessen. Mit diesen Daten wird dann bis zum Zustand reiner Wärmeleitung extrapoliert.

Die gemessene kritische Rayleigh-Zahl für die horizontale Luftschicht stimmt bis auf 1% mit dem bekannten Wert von 1708 überein.

Für die vertikale Luftschicht beträgt der Wert der gemessenen kritischen Grashof-Zahl 11 000 ± 510, in sehr guter Übereinstimmung mit dem von Unny angegebenen Wert von 11 024. Für geneigte Luftschichten sind die Ergebnisse für Neigungswinkel, gemessen zur Horizontalen, von 15, 30, 45, 60, 75, 80 und 85° angegeben.

Die Ergebnisse stimmen mit den Werten von Unny und Hart bis auf eine maximale Abweichung von 20% überein. Die Nusselt-Zahlen in der Nähe der Stabilitätsgrenze sind ebenfalls angegeben.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ УСТОЙЧИВОСТИ ДИФФЕРЕНЦИАЛЬНО НАГРЕВАЕМЫХ НАКЛОННЫХ ПЛАСТОВ ВОЗДУХА

Аннотация—В статье приводится экспериментальное определение критических чисел Релея, ниже которых обеспечивается устойчивость режма теплопроводности в горизонтальных, вертикальных и наклонных слоях воздуха. Экспериментальный метод состоит в измерении теплового потока вблизи области неустойчивости и экстраполировании этих данных на случай чистой теплопроводности. Измеренное критическое число Рейнольдса для горизонтального слоя составляет 1% от общепринятого значения (1708). В случае вертикального слоя измеренное критическое число Грасгофа находится в пределах 11000 ± 510, что очень близко соответствует значению 11024, предсказанному Юнни. Для наклонных слоев результаты представлены для углов наклона 15, 30, 45, 60, 80, 75 и 85°. Они соответствуют расчетам Юнни-Харта с максимальным отклонением до 20%. Приводятся также значения числа Нусскльта в непосредственной близости от области нестабильности.